Collision-based Computing with Cellular Automata

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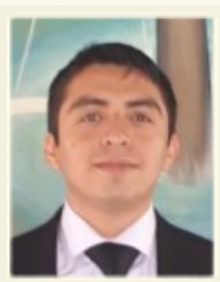
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Complex cellular automata a way to unconventional computing

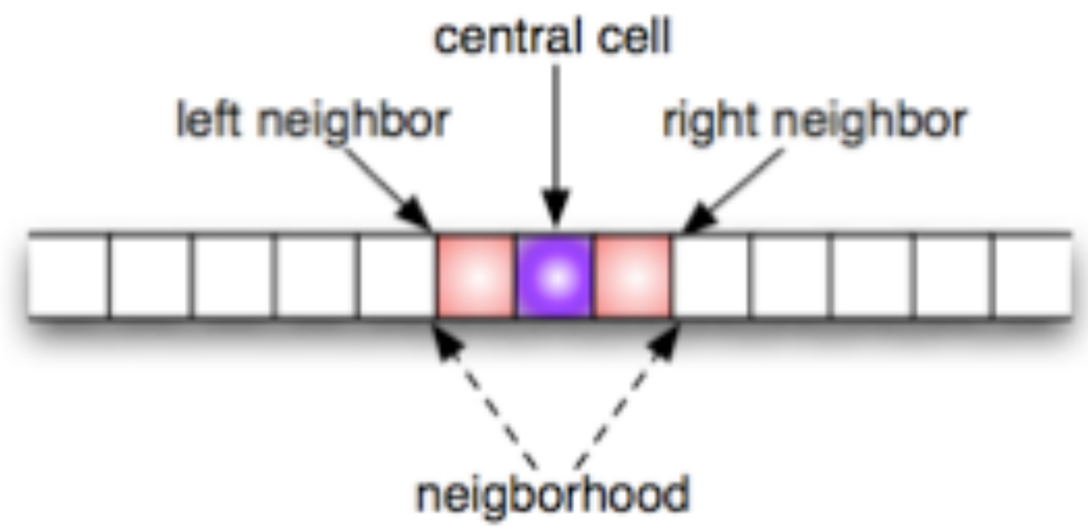
Today, a "computer", without further qualifications, denotes a rather well-specified kind of object; we'll consider a computer "non-conventional" if its physical substrate or its organization significantly depart from this de facto norm. [Toffoli 1998]

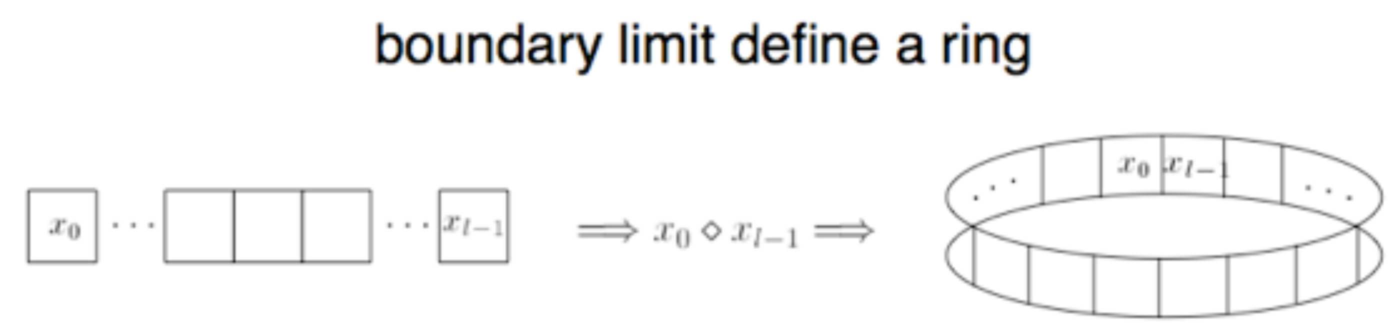
Unconventional computers shall exploit molecular computing level, to increase the power of computation, velocity, and storage. Actually, the term about of *unconventional computing* or *natural computing* have a number of directions:

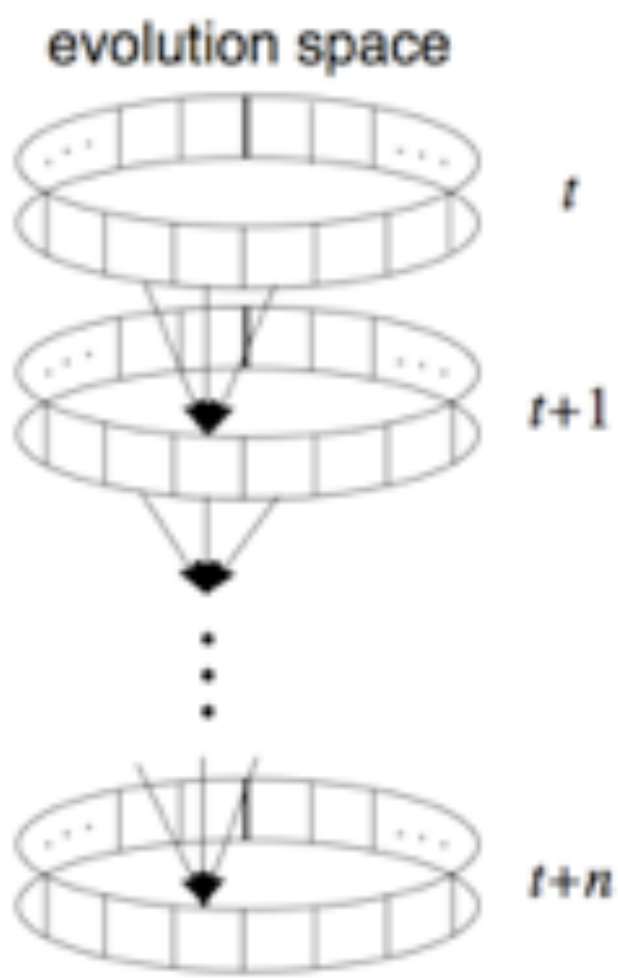
- quantum computing
- DNA computing
- reaction-diffusion computing
- reversible computing
- tiling (pattern) computing
- origami computing
- Pysarum computing
- swarm computing

- Toffoli, T. (1998) **Non-Conventional Computers**, *Encyclopedia of Electrical and Electronics Engineering* (John Webster Ed.), 14:455-471, Wiley & Sons.
- Mills, J.W. (2008) **The Nature of the Extended Analog Computer**, *Physica D*, 237(9):1235-1256.
- Fredkin, E. & Toffoli, T. (2002) **Design Principles for Achieving High-Performance Submicron Digital Technologies**, In: *Collision-Based Computing*, A. Adamatzky (Ed.), Springer, chapter 2, 27-46.

Dynamics in one dimension







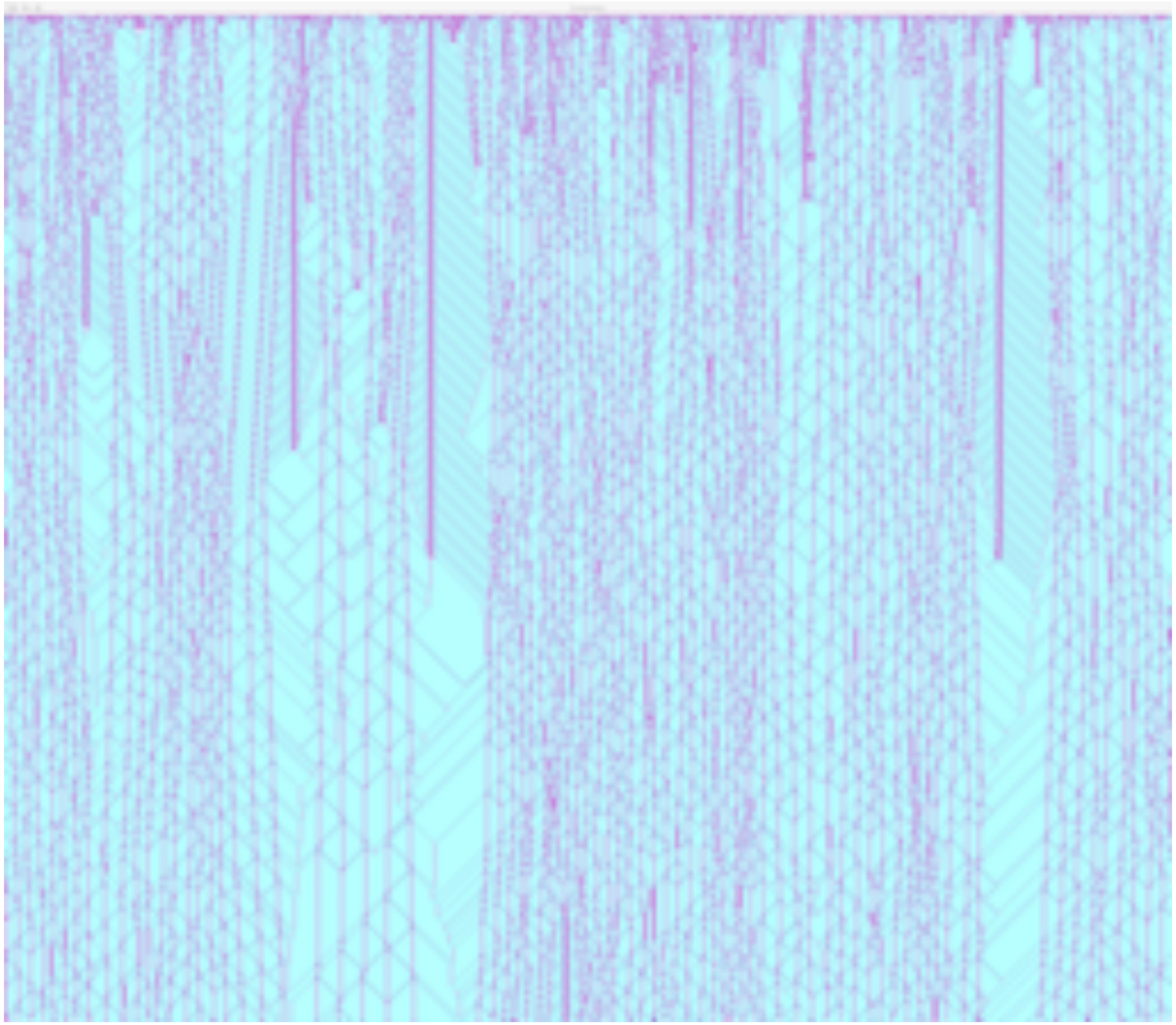
Elemental CA (ECA) is defined as follows:

- $\Sigma = \{0,1\}$
- $\mu = (x_{+1}, x_0, x_{-1})$ such that $x \in \Sigma$
- $\phi : \Sigma^3 \to \Sigma$
- $\mu = \{c_0 \mid x \in \Sigma\}$ the initial condition is the first ring with $t = 0$

Elementary cellular automaton rule 54

Some interesting points in rule 54:

- Artificial life
- Complex systems
- Logical computation
- Garden of Eden configurations
- Symmetric evolutions
- Guns emerge from random conditions



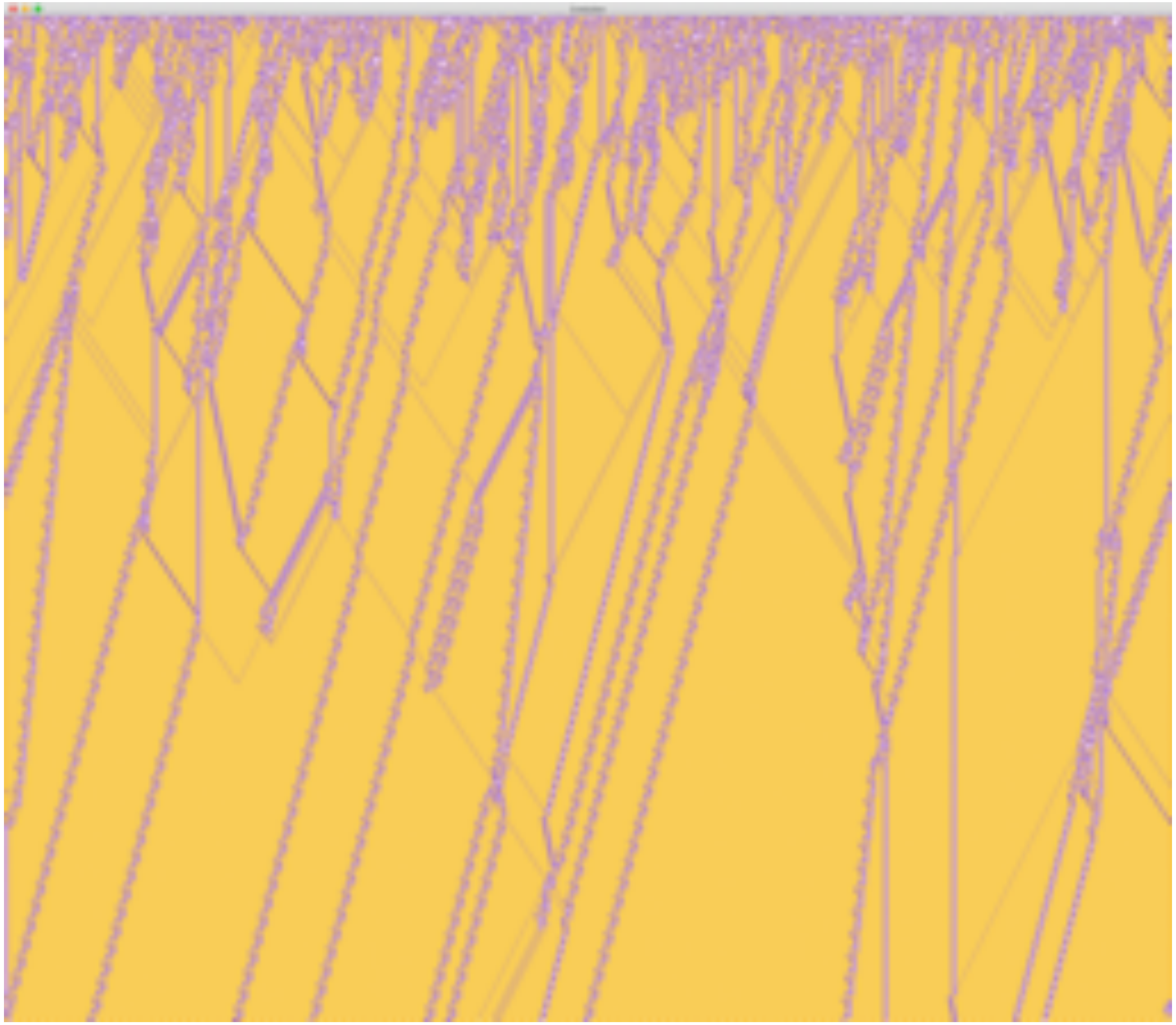
1900 cells x 1640 times

- Boccara, N., Nasser, J. & Roger, M. (1991) **Particle like structures and their interactions in spatio-temporal patterns generated by one-dimensional deterministic cellular automaton rules,** Physical Review A 44(2), 866-875.
- Hanson, J.E. & Crutchfield, J.P. (1997) **Computacional Mechanics of Cellular Automata: An Example**, Physics D 103(1-4), 169-189.
- Martin, B. (2000) **A Group Interpretation of Particles Generated by One-Dimensional Cellular Automaton, Wolfram's Rule 54**, Int. J. of Modern Physics C 11(1), 101-123.
- Martínez, G.J., Adamatzky, A. & McIntosh, H.V. (2006) **Phenomenology of glider collisions in cellular automaton Rule 54 and associated logical gates**, Chaos, Fractals and Solitons 28, 100-111.
- Guan, J. (2012) **Complex Dynamics of the Elementary Cellular Automaton Rule 54**, International Journal of Modern Physics C 23(7), 1250052.

Elementary cellular automaton rule 110

Some interesting points in rule 110:

- Artificial life
- Complex systems
- Universal computation
- Garden of Eden configurations
- Asymmetric evolutions
- Extendible gliders



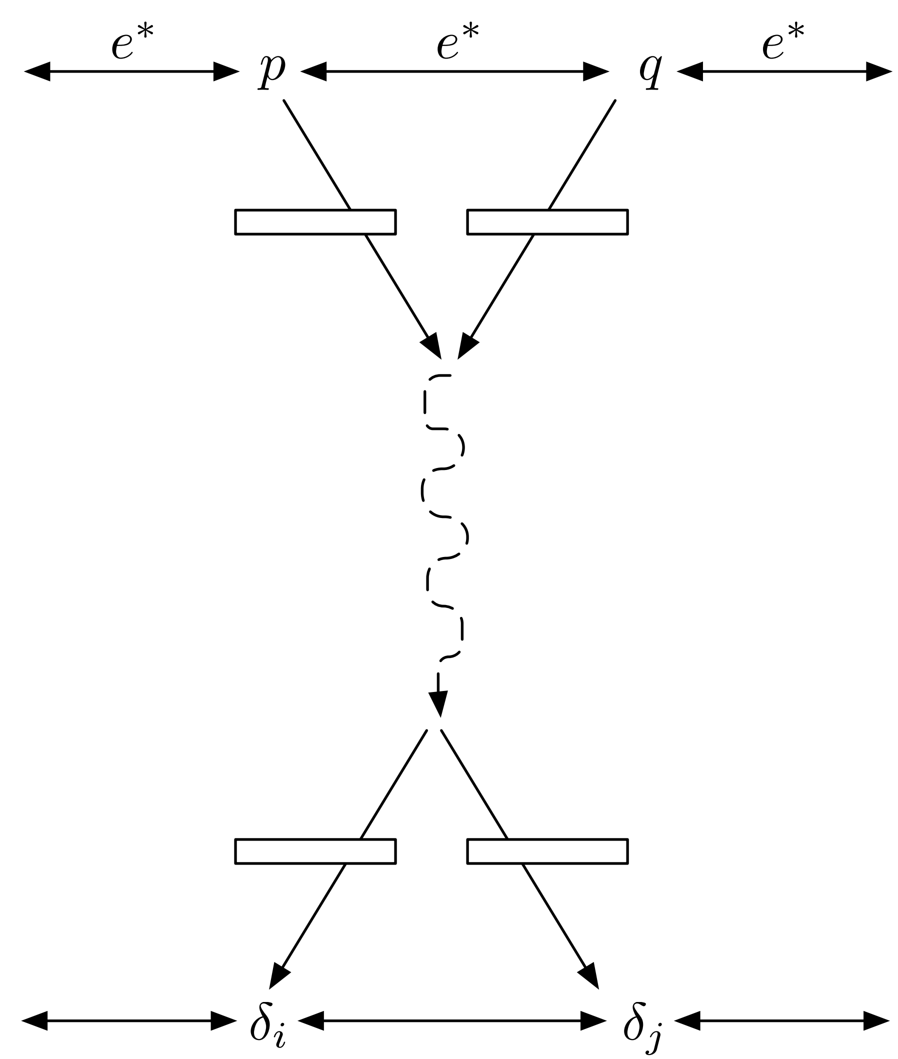
1900 cells x 1640 times

- Wolfram, S. (1994) **Cellular Automata and Complexity: collected papers**, Addison-Wesley Publishing Company.
- Cook, M. (1999) **Introduction to the activity of rule 110** (copyright 1994-1998 Matthew Cook), http://w3.datanet.hu/~cook/Workshop/CellAut/Elementary/Rule110/110pics.html, January.
- McIntosh, H.V. (1999) **Rule 110 as it relates to the presence of gliders**, http://delta.cs.cinvestav.mx/~mcintosh/oldweb/pautomata.html
- Martínez, G.J., McIntosh, H.V. & Mora, J.C.S.T. (2006) **Gliders in Rule 110**, International Journal of Unconventional Computing 2(1), 1-49.
- Mora, J.C.S.T., Martínez, G.J., Romero, N.H. & Marín, J.M. (2010) **Elementary cellular automaton Rule 110 explained as a block substitution**, Computing 88, 193-205.

Particles are strings

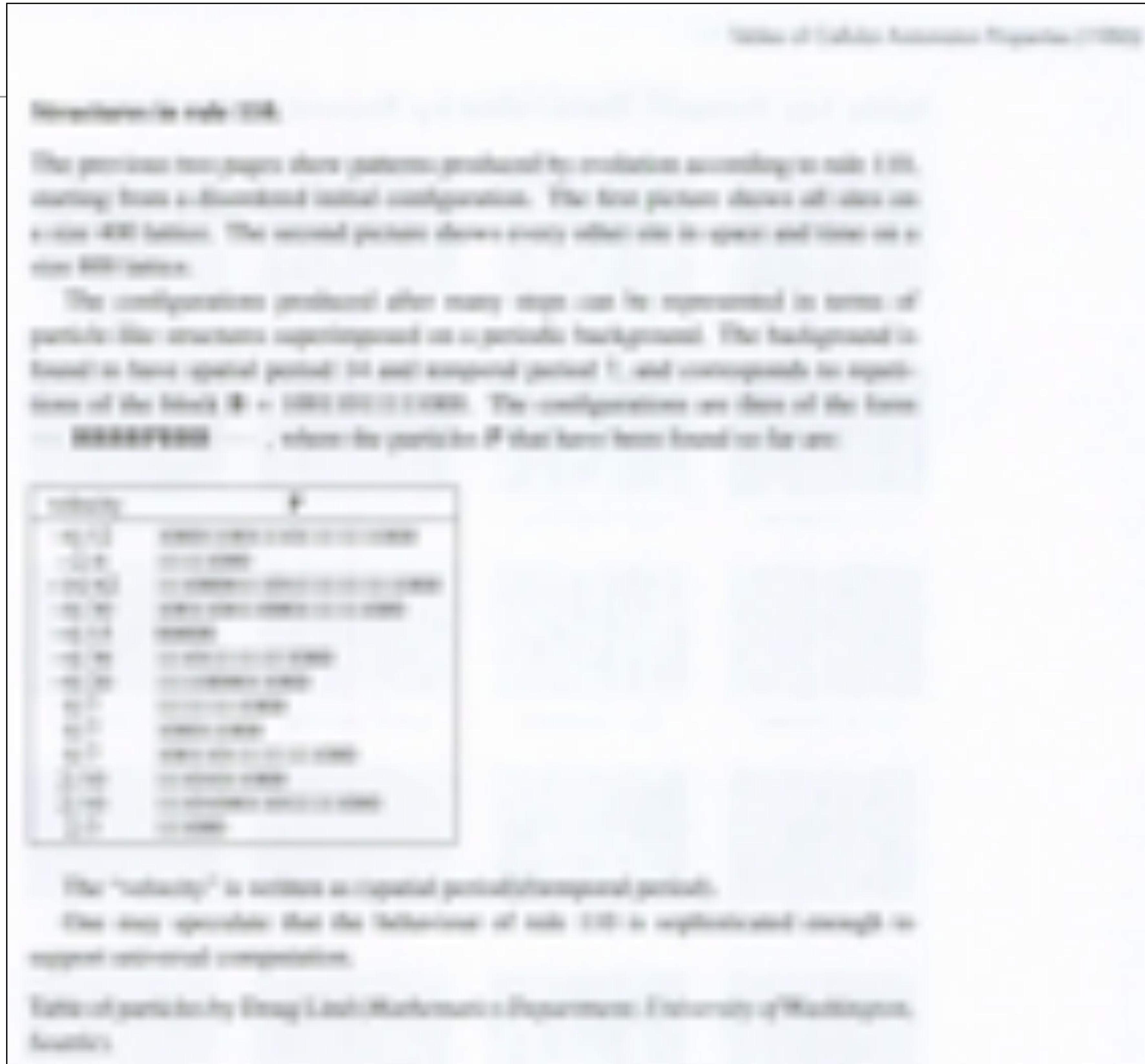
The formal languages theory provides a way to study sets of chains from a finite alphabet. The languages can be seen as inputs for some classes of machines or as the final result from a typesetter substitution system i.e., a generative grammar into the Chomsky's classification. This way, following a variation of a Feynman diagram hence we can represent collisions between particles in one-dimensional cellular automata as follows.

- p, q, δ – particles
- e – periodic background
- $\square$ – phase
- $f(p,q) \rightarrow \delta$



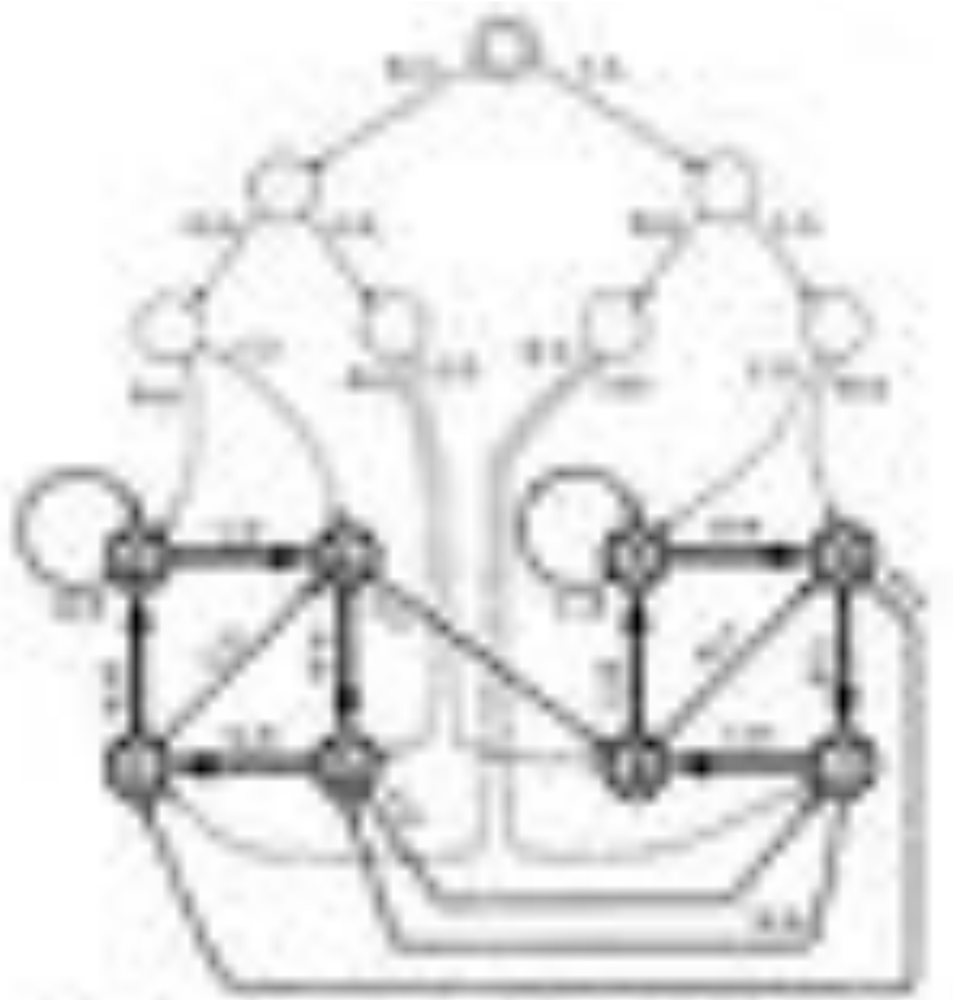
- Hurd, L.P. (1987) **Formal Language Characterizations of Cellular Automaton Limit Sets**, *Complex Systems 1*, 69-80.
- Wolfram, S. (1984) "**Computation Theory on Cellular Automata**," *Communication in Mathematical Physics 96*, 15-57.
- Hopcroft, J.E. & Ullman, J.D. (1987) ***Introduction to Automata Theory Languajes, and Computation***, Addison-Wesley Publishing Company.
- Collaborative Research Center SFB 676, **Particles, Strings, and the Early Universe**, Universität Hamburg , http://wwwiexp.desy.de/sfb676/

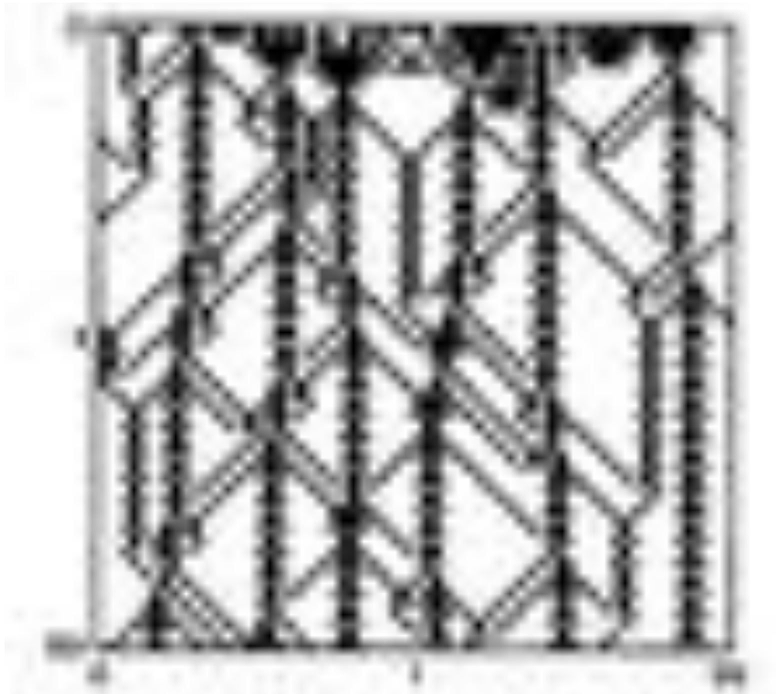
Particles are strings: Doug Lind starts this representation in ECA from 1986



Lind, D. (1994) **Structures in rule 110**, In: *Cellular Automata and Complexity*, Stephen Wolfram, Table of Properties, page 577, http://www.stephenwolfram.com/publications/articles/ca/86-caappendix/16/text.html

Particles are strings and filters: James Hanson and James Crutchfield describing finite state machines in ECA from 1997





Hanson, J.E. & Crutchfield, J.P. (1997) **Computational mechanics of cellular automata: An example**, *Physica D*, 103(1-4), 169-189.

Particles are strings: Harold McIntosh stablished that the problem of rule 110 is a problem of tiles in 1998

1.4.4 ether crystallography

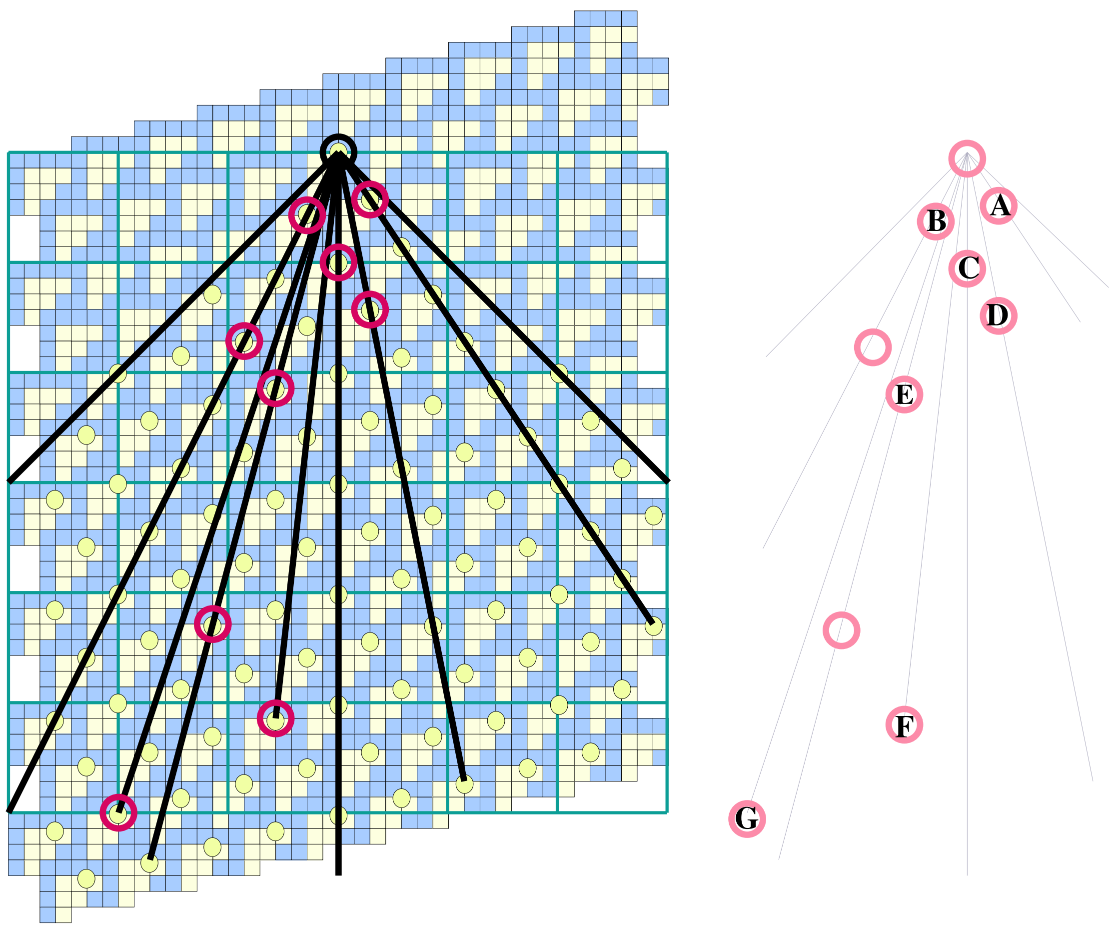


Figure 1.13: The locations of Cook's gliders relative to the ether lattice. The two barred gliders sit lower on the same velocity lines as the unbarred gliders. Small circles on the T3 mosaic show possible positions of compatible gliders, but they could be impossible, duplicates, or so far undiscovered.

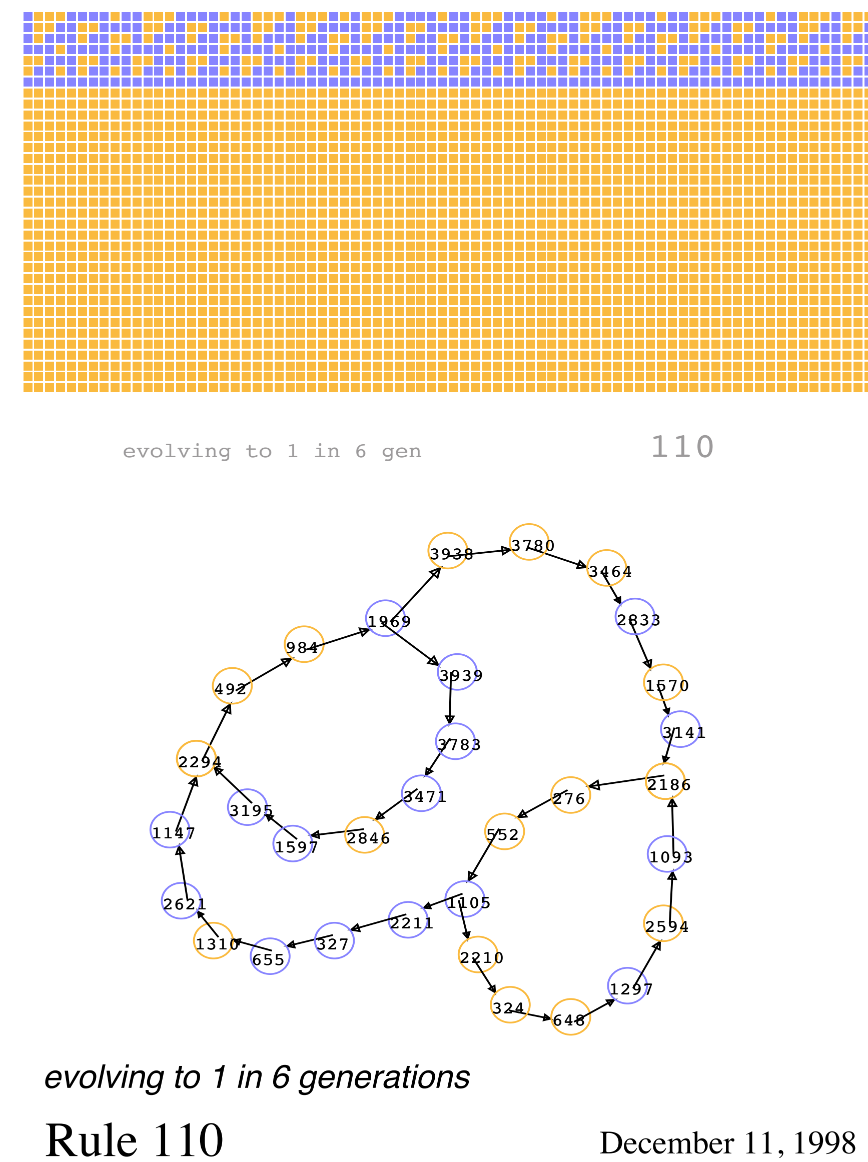


Figure 1.22: The de Bruijn diagram for evolution to the constant 1 after six generations.

McIntosh, H.V. (1999) **Rule 110 as it Relates to the Presence of Gliders.** Technical report UAP. http://delta.cs.cinvestav.mx/~mcintosh/cellularautomata/Papers.html

Particles as strings from finite states machines

For an one-dimensional cellular automaton of order (k,r), the *de Bruijn diagram* is defined as a directed graph with k^{2r} vertices and k^{2r+1} edges. The vertices are labeled with the elements of the alphabet of length 2r. An edge is directed from vertex i to vertex j, if and only if, the 2r-1 final symbols of i are the same that the 2r-1 initial ones in j forming a neighbourhood of 2r+1 states represented by $i \lozenge j$. In this case, the edge connecting i to j is labeled with $\phi(i \lozenge j)$.

The connection matrix M corresponding with the de Bruijn diagram is as follows:

$$M_{i,j} = \begin{cases} 1 & \text{if } j = ki, ki+1, \ldots, ki+k-1 \pmod{k^{2r}} \\ 0 & \text{in other case} \end{cases}$$

Basins of attraction or cycle diagrams calculate attractors in a dynamical system, as was extensively studied by Andrew Wuensche in CA and random Boolean networks. Given a sequence of cells *x_i* we define a configuration *c* of the system. An evolution is represented by a sequence of configurations *c*_0, *c*_1, *c*_2, …, *c_{m*-1*}* given by the global mapping,

$$\Phi : \Sigma^n \to \Sigma^n$$

and the global relation is given for the next function between configurations,

- McIntosh, H.V. (1991) **Linear cellular automata via de Bruijn diagrams**, http://delta.cs.cinvestav.mx/~mcintosh/oldweb/pautomata.html
- Sutner, K. (1991) **De Bruijn Graphs and Linear Cellular Automata**, Complex Systems 5(1) 19-30.
- Wuensche, A. & Lesser, M. (1992) **Global Dynamics of Cellular Automata,** Addison-Wesley Publishing Company.
- Voorhees, B.V. (1996) ***Computational analysis of one-dimensional cellular automata***, World Scientific Series on Nonlinear Science, Series A, Vol. 15.
- McIntosh, H.V. (2009) ***One Dimensional Cellular Automata***, Luniver Press.

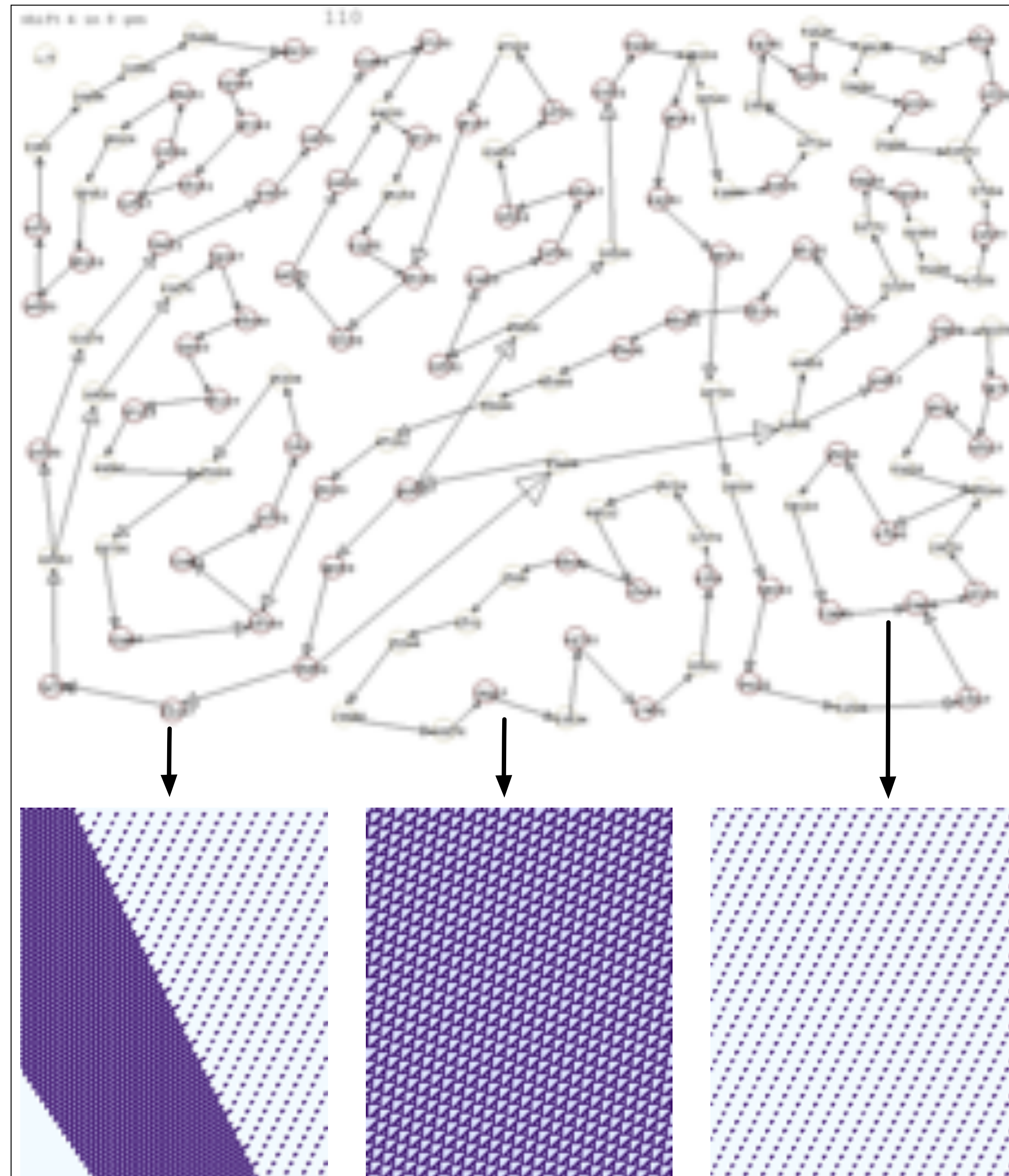
Particles as strings from finite states machines

De Bruijn diagram (displacement 4, period 8) calculating non-stationary particles in rule 110. The left evolution displays a *fuse pattern* produced by two particles colliding and both annihilated. The center evolution displays a periodic pattern and the right evolution displays particles with displacement to the left.

$$N = \{61166, 56799, 48059, 30583, 61167, 56703, 48062, 30589, 61178, 56821, 48107, 30679, 61369, 57183, 48830, 32125, 64250, 62965, 60395, 55255, 44975, 24415\}$$

$$((0111)^* + 11(01011111)^*)^*$$

It is an expression to codify *A* and *B* particles.



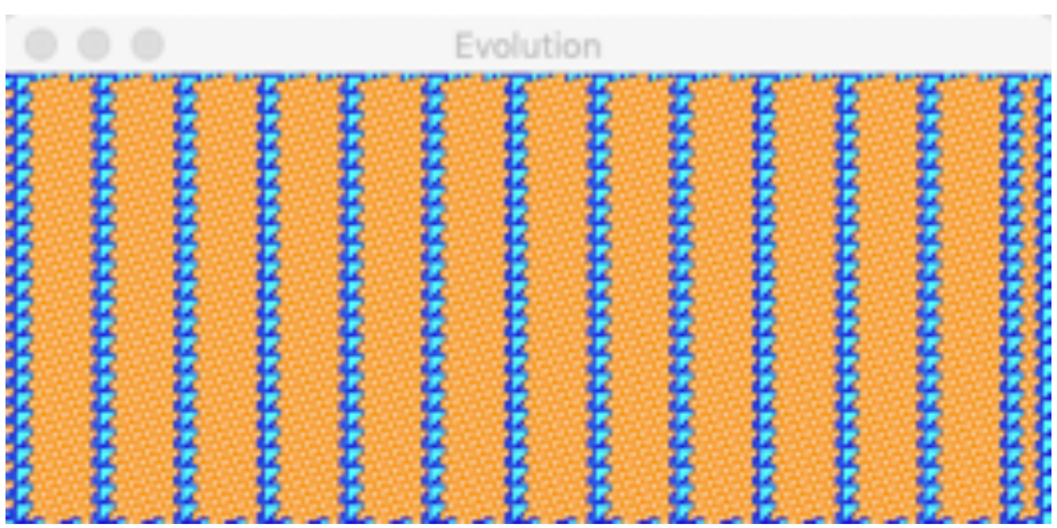
Martínez, G.J., Adamatzky, A., Chen, B., Chen, F. & Mora, J.C.S.T. (2018). **Simple networks on complex cellular automata: from de Bruijn diagrams to jump-graphs**. In: Evolutionary Algorithms, Swarm Dynamics and Complex Networks (pp. 241-264). Springer, Berlin, Heidelberg.

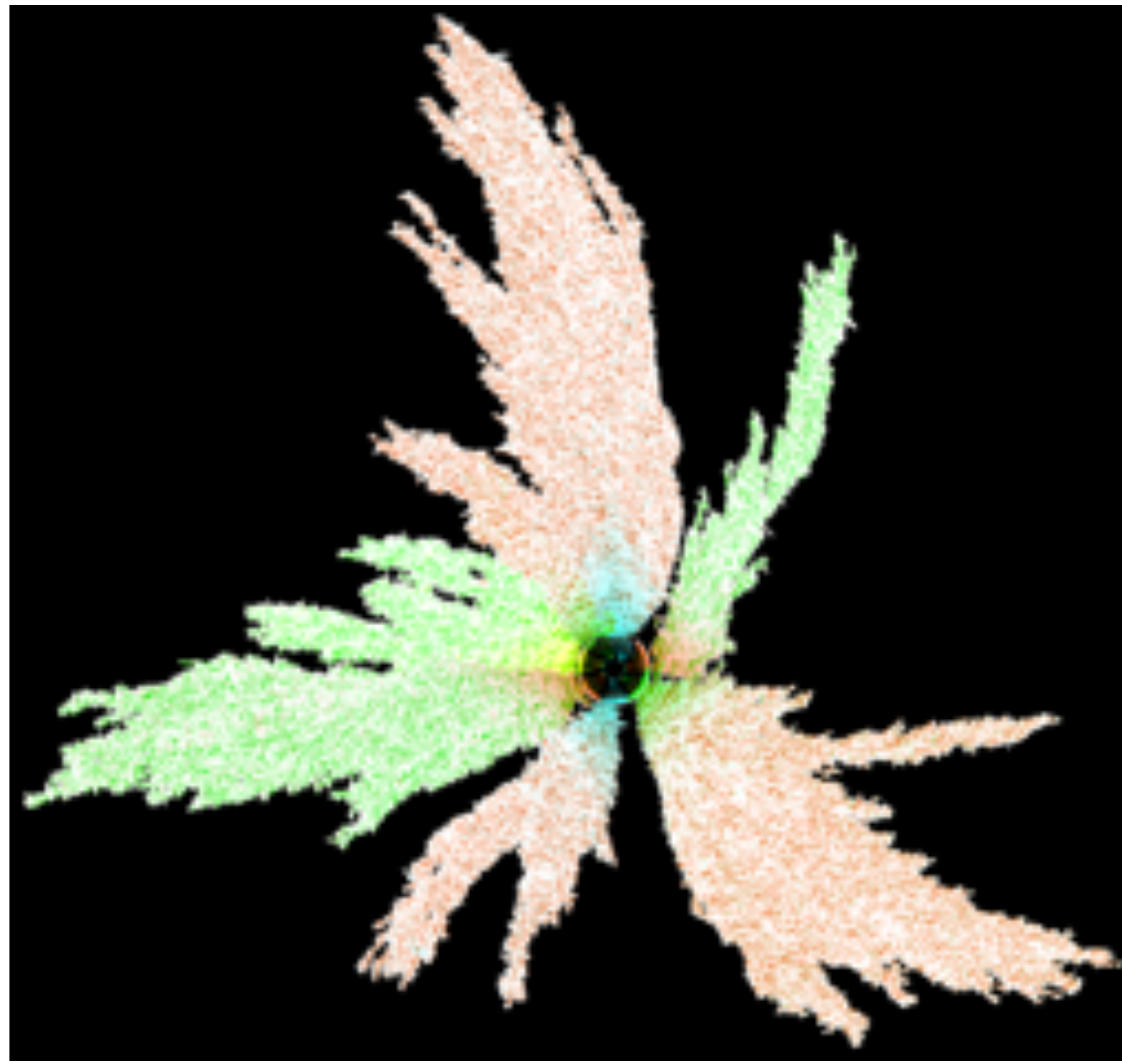
Particles as strings from finite states machines

Attractor length 31, period 7, with a mass of 5.487x10⁷ configurations for rule 110. This attractor is in a field of 6,326 basins.

$$
\begin{aligned}
w_0 &= 0110111111110001001101111100010\\
w_1 &= 1111100000010011011111000100110\\
w_2 &= 1000100000110111111000100110111\\
w_3 &= 1001100000111110001001101111000\\
w_4 &= 1011100011000100110111110001001\\
w_5 &= 1110100111001101111100010011011\\
w_6 &= 0011101101011111000100110111110
\end{aligned}
$$

They are expressions to codify stationary (*C*s) particles.



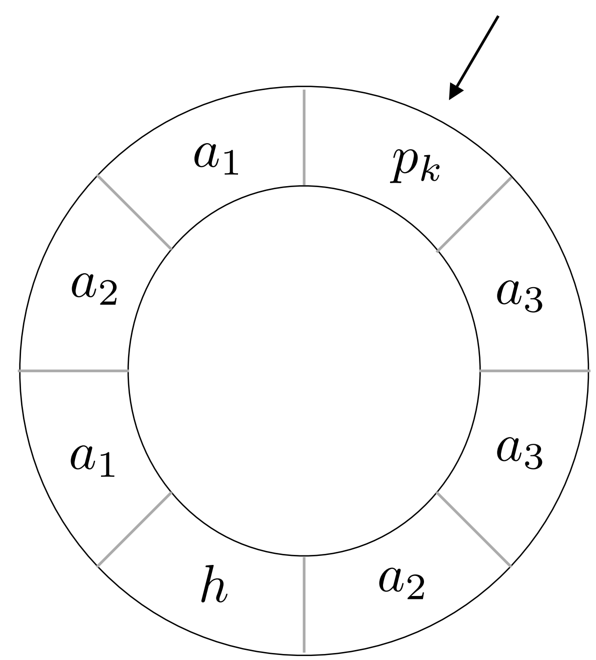


Martínez, G.J., Adamatzky, A., Chen, B., Chen, F. & Mora, J.C.S.T. (2018). **Simple networks on complex cellular automata: from de Bruijn diagrams to jump-graphs**. In: Evolutionary Algorithms, Swarm Dynamics and Complex Networks (pp. 241-264). Springer, Berlin, Heidelberg.

Circular computation

We have two important previous results in computer science theory to think about of circular computation. Arbib presents a circular Turing machine in 1962 and Kudlek and Rogozhin presents circular Post machines in 2001.

- a_i, h : symbols
- h : the limit of the type
- p_k : state
- $\rightarrow$: head



Important features: (1) label and limiting the end of the type, (2) the movement is turning the type, (3) the type can introduce new squares.

- Arbib, M.A. (1962) **Monogenic Normal Systems are Universal**, Monogenic normal systems are universal. Journal of the Australian Mathematical Society, 3(3) 301-306.
- Kudlek, M. & Rogozhin, Y. (2001) **Small Universal Circular Post Machines,** Computer Science Journal of Moldova 9(1) 34-52.

Symbol super colliders (Tommaso Toffoli, 2002)

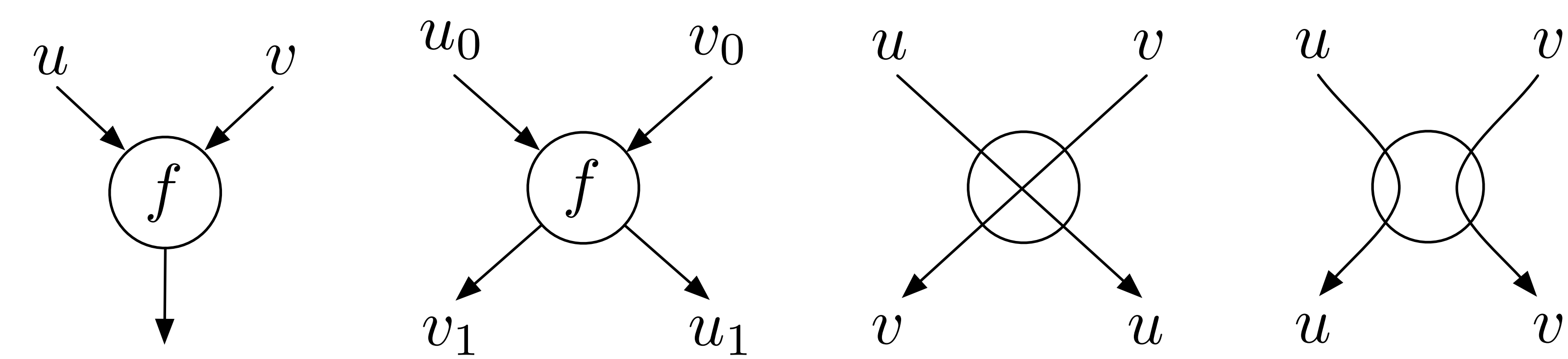
In 2002, Toffoli exposed the concept of “symbol super collider”. To map Toffoli's supercollider onto a one-dimensional cellular automata we use the notion of an idealized particle $p \in Z^+$ (without energy and potential energy). The particle p is represented by a binary string of cell states. Typically, we can find all types of particles manifest in cellular automata particles, including positive p^+, negative p^-, and neutral p^0 displacements, and also composite particles assembled from elementary particles.

(a) $f(u, v)$ is a product of one collision

(b) $f(u, v) = u + v$ union

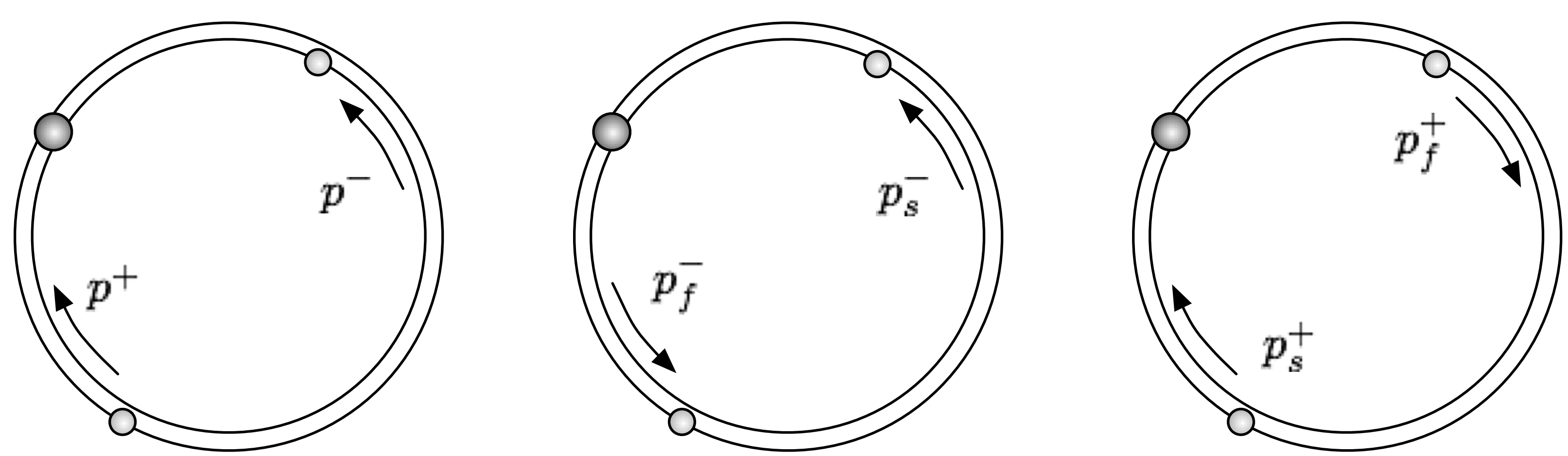
(c) $f_i(u, v) \mapsto (u, v)$ identity

(d) $f_r(u, v) \mapsto (v, u)$ reflection



(a) (b) (c) (d)

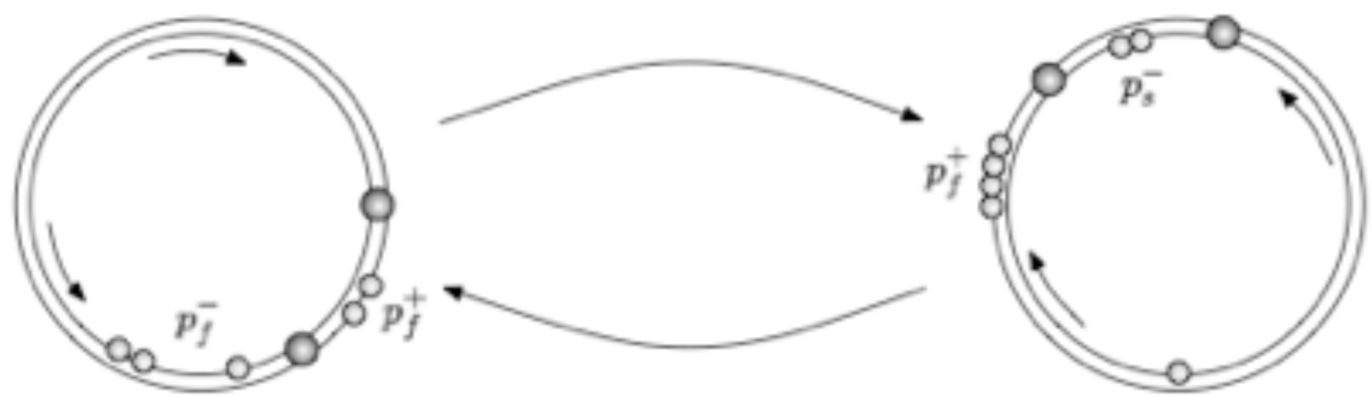
Schemes of ballistic collisions between particles evolving in *cyclotrons*. Gray circles represent the contact point of collision.



Toffoli, T. (2002) **Symbol Super Colliders**, In: Adamatzky, A. (Ed.) *Collision-Based Computing*, Springer, chapter 1, pages 1-22.

Symbol super colliders

Transition between two beam routing synchronizing multiple reactions. When the first set of collisions are done a new beam routing is defined with other particles, so that when the second set of collisions is done then one returns to the initial condition of the first beam, constructing a meta-glider or mesh in Rule 110.



In this way, we can design more complex constructions synchronizing multiple collisions with a diversity of speeds and phases on different particles. Figure displays a more sophisticated beam routing design, connecting two of beams and then creating a new beam routing diagram where edges represent a change of particles and collisions contact point on ECA Rule 110. In such a transition, a number of new particles emerge and collide to return to the first beam, thus oscillating between two beam routing forever.

$$p_A^+, p_A^+ \leftrightarrow p_{\bar{B}}^-, p_B^-, p_B^-$$

changing to the set of particles (second beam routing):

$$p_{A^4}^+ \leftrightarrow p_E^+, p_{\bar{E}}^+$$

defining two beam routing connected by a transition of collisions as:

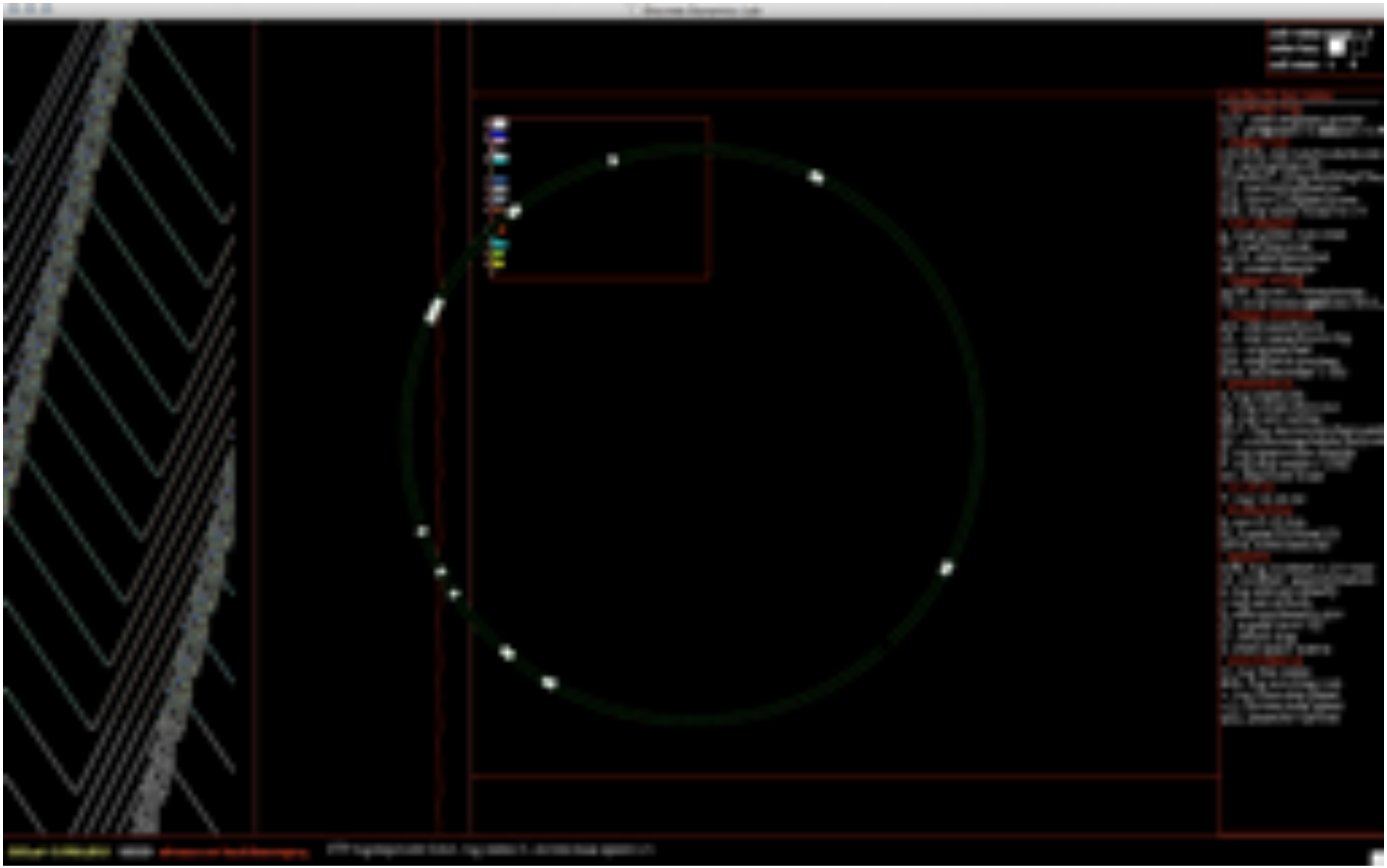
$$(p_A^+, p_A^+ \leftrightarrow p_{\bar{B}}^-, p_B^-, p_B^-) \rightarrow (p_{A^4}^+ \leftrightarrow p_E^+, p_{\bar{E}}^+), \text{ and}$$

$$(p_{A^4}^+ \leftrightarrow p_E^+, p_{\bar{E}}^+) \rightarrow (p_A^+, p_A^+ \leftrightarrow p_{\bar{B}}^-, p_B^-, p_{\bar{B}}^-).$$

DDLab evolving cellular automata as rings (cyclotrons)

Cyclotrons are the first stage where we can see periodic collisions or simple dynamics of particles traveling around the ring.

A multiple collision between eight particles we can produce a glider gun in rule 110. In this simulation using DDLab, we can see the typical two dimensional representation a nd at the center the cyclotronic view, with the periodic background filtered.

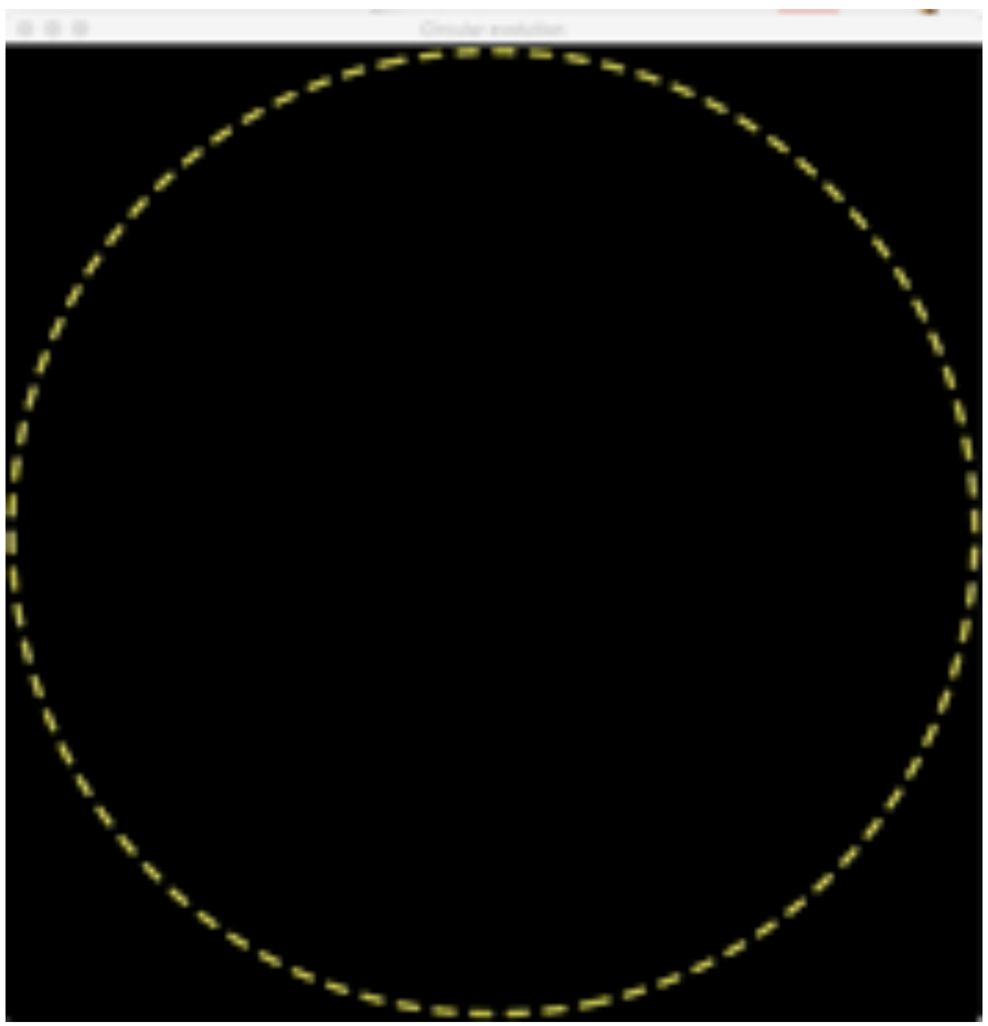


Construction of patterns by synchronization of multiple collisions in rule 54

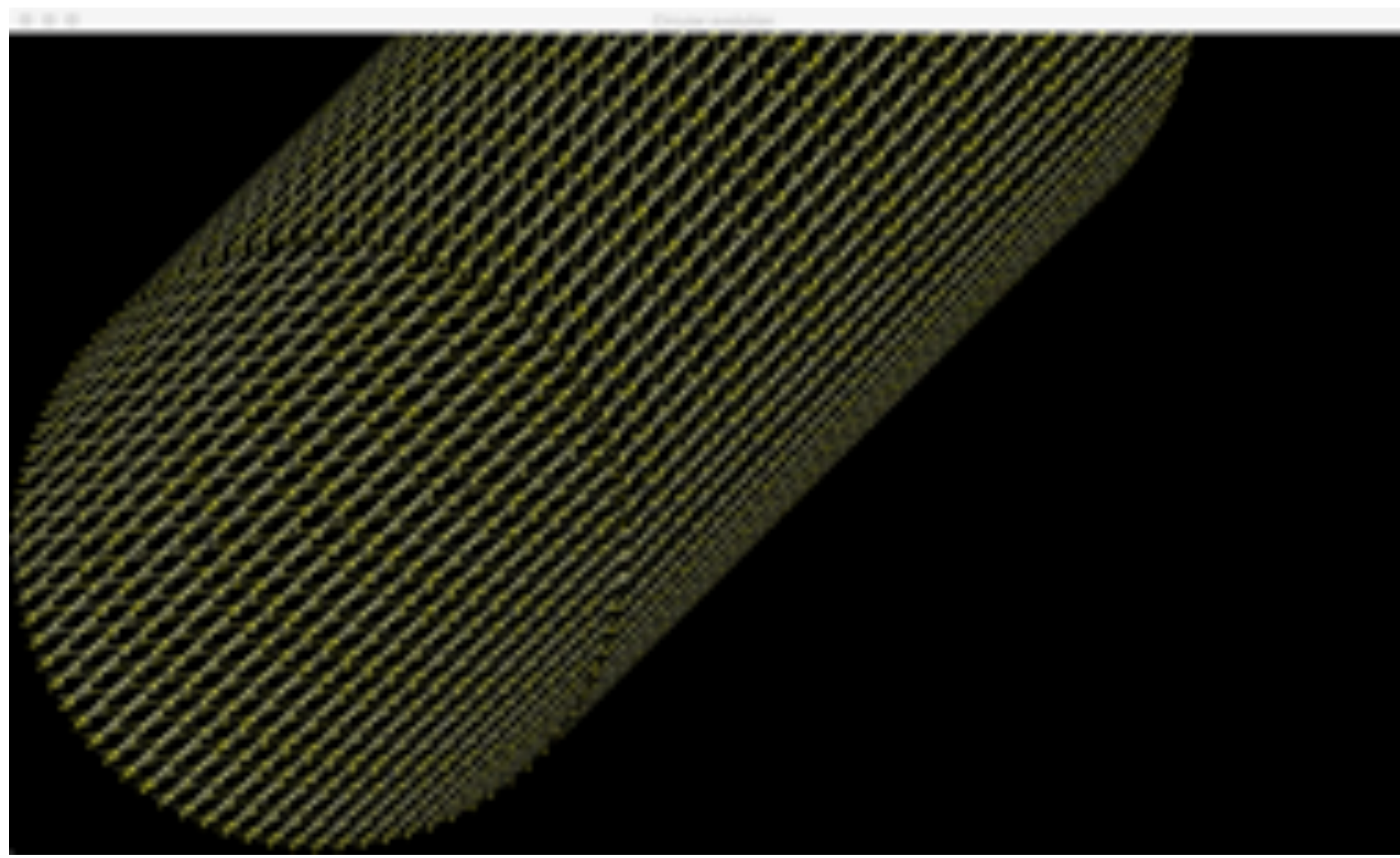
Using a cyclotron we can design patterns with other views. This simulation starts with an initial condition of 3,214 cells codifying 216 particles in rule 54. The reaction is a triple collision controller by two basic interactions:

$$f(g_e, \overleftarrow{w}) \rightarrow \overleftarrow{w}, g_e, 2\overrightarrow{w}\ ;\ f(2\overrightarrow{w}, \overleftarrow{w}) \rightarrow \overleftarrow{w}$$

The reaction starts with a negative particle *w* colliding versus a stationary particle *g* yielding a negative particle plus two positive particles *w*. But the first negative particle finds these new pairs of positive particles and annihilate them. Therefore, we can codify these particles with the next expression: $(\overleftarrow{w}\text{-}2e\text{-}g_e(\mathrm{A},\mathrm{f}_1)\text{-}2e\text{-}\overleftarrow{w})^*$.

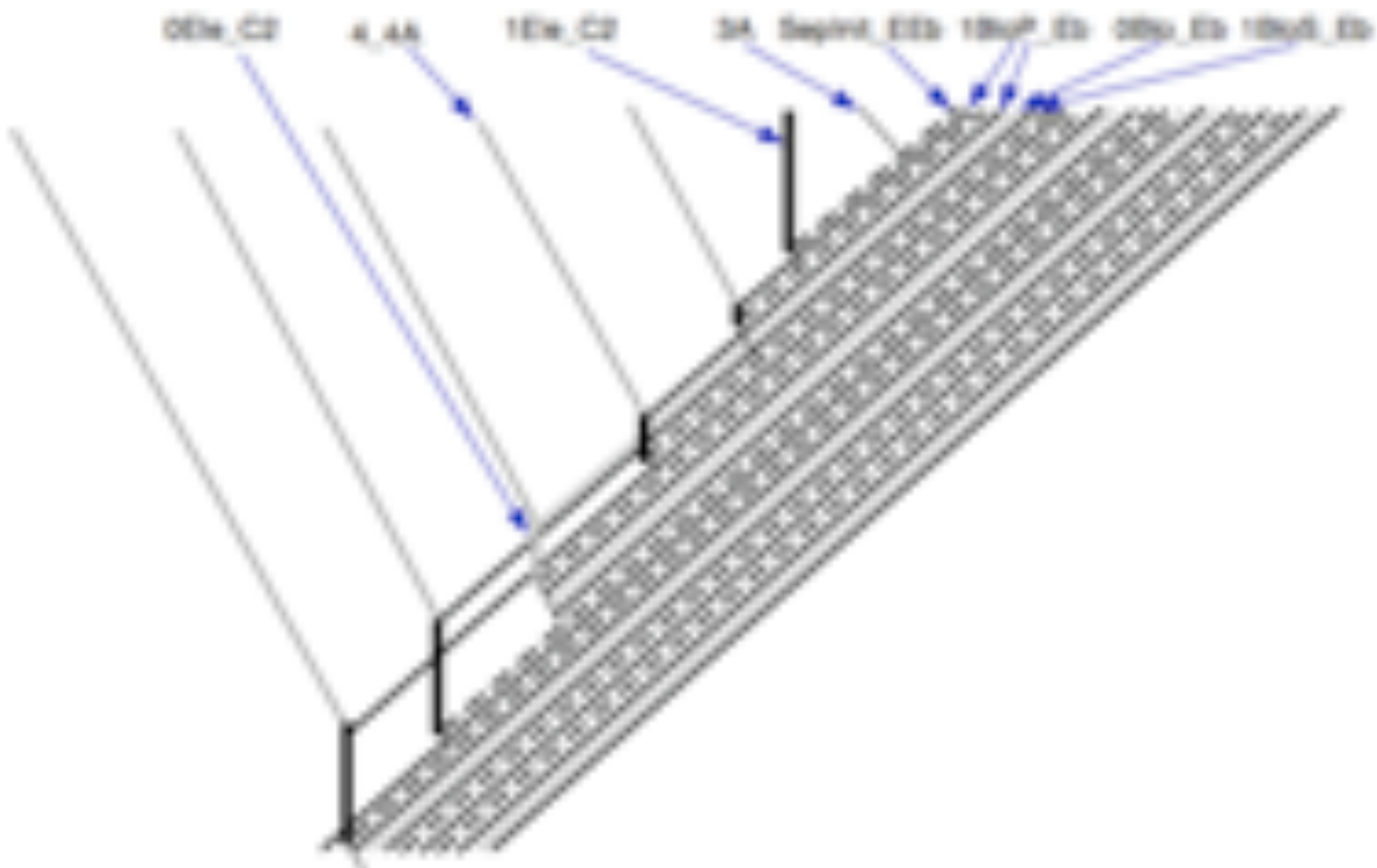


circular



three-dimensional projection

Cyclic tag systems in a finite codification



A diagram of a cyclic tag system (CTS) working in rule 110

Beam routing codification representing package of particles which reproduces a CTS in rule 110

- Cook, M. (2004) **Universality in Elementary Cellular Automata**. Complex Systems 15(1), 1-40.
- Cook, M. (2008) **A Concrete View of Rule 110 Computation**. In: The Complexity of Simple Programs, T. Neary, D.Woods, A.K. Seda and N. Murphy (Eds.), 31-55.
- Wolfram, S. (2002) **A New Kind of Science**, Wolfram Media, Inc., Champaign, Illinois.
- Neary, T. & Woods, D. (2006) **P-completeness of cellular automaton Rule 110**. Lecture Notes in Computer Science 4051, 132-143.
- Martínez, G.J., McIntosh, H.V., Mora, J.C.S.T. & Vergara, S.V.C. (2011) **Reproducing the cyclic tag system developed by Matthew Cook with Rule 110 using the phases f1_1**, *Journal of Cellular Automata 6(2-3)*, 121-161.

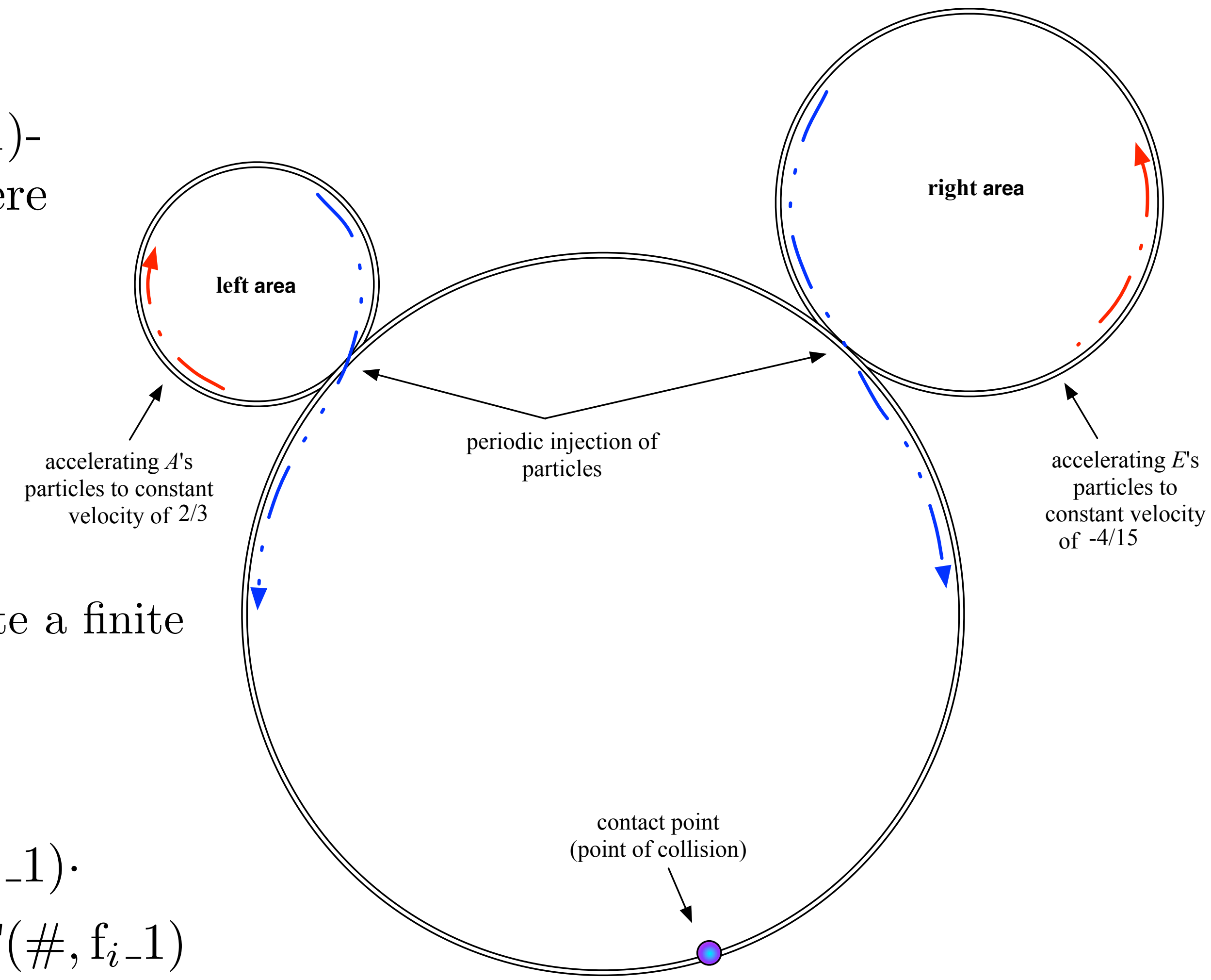
Cyclic tag systems in a finite codification as a collider

This way, the cyclic tag system working in rule 110 can be simplified as follows:

left: $\{649e\text{-}4\_A^4(\text{F}\_i)\}^*$, for $1 \leq i \leq 3$ in sequential order

center: $246e\text{-}1\text{Ele\_C2}(\text{A},\text{f}_1\_1)\text{-}e\text{-}A^3(\text{f}_1\_1)$

right: $\{\text{SepInit\_}E\bar{E}(\#,\text{f}_i\_1)\text{-}1\text{BloP\_}\bar{E}(\#,\text{f}_i\_1)\text{-SepInit\_}E\bar{E}(\#,\text{f}_i\_1)\text{-}1\text{BloP\_}\bar{E}(\#,\text{f}_i\_1)\text{-}0\text{Blo\_}\bar{E}(\#,\text{f}_i\_1)\text{-}1\text{BloS\_}\bar{E}(\#,\text{f}_i\_1)\}^*$ (where $1 \leq i \leq 4$ and $\#$ represents a particular phase).



A diagram of a cyclic tag system (CTS) working in rule 110

This way, we have that the string $w_{CTSR110}$ is a word able to simulate a finite state machine into a cellular automata collider.

$$w_{CTSR110} = (649e \cdot 4\_A^4(\text{F}\_i))^* \cdot (246e \cdot 1\text{Ele\_}C_2(\text{A},\text{f}_1\_1) \cdot e \cdot A^3(\text{f}_1\_1)) \cdot (\text{SepInit\_}E\bar{E}(\#,\text{f}_i\_1) \cdot 1\text{BloP\_}\bar{E}(\#,\text{f}_i\_1) \cdot \text{SepInit\_}E\bar{E}(\#,\text{f}_i\_1) \cdot 1\text{BloP\_}\bar{E}(\#,\text{f}_i\_1) \cdot 0\text{Blo\_}\bar{E}(\#,\text{f}_i\_1) \cdot 1\text{BloS\_}\bar{E}(\#,\text{f}_i\_1))^*.$$

Genaro J. Martínez, Andrew Adamatzky, Harold V. McIntosh (2017) **A Computation in a Cellular Automaton Collider Rule 110**, In: *Advances in Unconventional Computing: Volume I Theory*, A. Adamatzky (Ed). chapter 15, pages 391-428.

Final remarks

Complex ECA rules with different capacities explored with cyclotrons.

rule	class	particle	particle^n	slopes	gun	gun^n	soliton	complex with memory	fractals
41	4	yes	no	+	no	no	no	yes	no
54	4	yes	no	-,+,s	yes	yes	yes	yes	no
106	4	yes	no	-	no	no	no	yes	no
110	4	yes	yes	-,+,s	yes	yes	yes	yes	no
22	3	yes	no	-,+	no	no	no	yes	yes
126	3	yes	no	-,+,s	no	no	no	yes	yes
26	2	yes	no	-,+	no	no	yes	yes	yes
62	2	yes	no	-,+	no	no	no	yes	no

- Martínez, G.J., Adamatzky, A., Hoffmann, R., Désérable, D. & Zelinka, I. (2019) **On Patterns and Dynamics of Rule 22 Cellular Automaton**. *Complex Systems* 28(2), 125-174.
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THE END

THANK YOU FOR YOUR KIND ATTENTION

Cellular Automata Repository
https://www.comunidad.escom.ipn.mx/genaro/CA_repository.html

Complex Cellular Automata Repository
https://www.comunidad.escom.ipn.mx/genaro/Complex_CA_repository.html

Cellular Automata Software
https://www.comunidad.escom.ipn.mx/genaro/Cellular_Automata_Repository/Software.html